How to solve Boundry Condition problems?!

① Method of images ☑

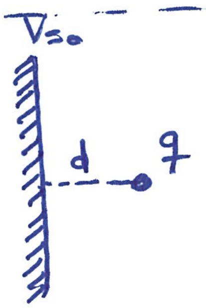
② Using orthogonal functions ☑

③ Conformal mapping ☒ [not here]

"General form of Green function"

$$G(x,x') = \frac{1}{|x-x'|} + F(x,x') \quad \text{where} \quad \nabla^2 F(x,x') = 0$$

$$\Phi(x) = \int_V \rho(x') G(x,x') d^3x' + \frac{1}{4\pi} \oint_S \left[G(x,x') \frac{\partial \Phi}{\partial n'} - \Phi(x') \frac{\partial G(x,x')}{\partial n'} \right] da'$$



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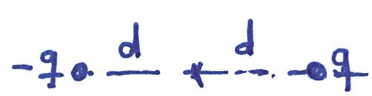
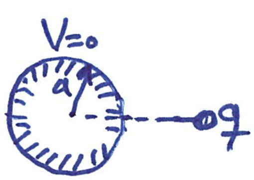


$$G(x,x') = \frac{1}{|x-d\hat{i}|} - \frac{1}{|x+d\hat{i}|}$$



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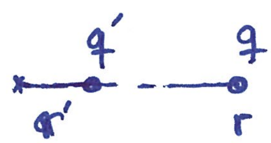
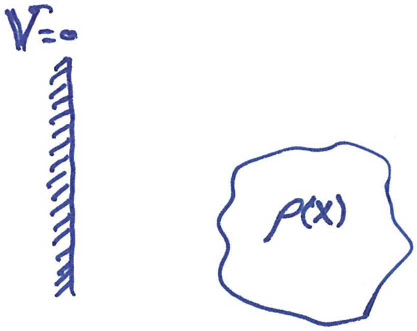


$$q' = \frac{a}{r} q$$

$$G(x,x') = \frac{1}{|x-r\hat{i}|} - \frac{a/r}{|x-r'\hat{i}|}$$

How to solve ?!



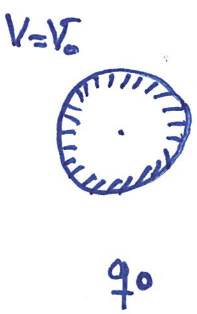
$$\Phi(x) = \int \rho(x') G(x,x')\, dx' + \frac{1}{4\pi} \oint_S \left[G(x,x') \frac{\partial \Phi}{\partial n'} - \phi \ldots \right] da'$$

we know at boundry $\phi(x') = 0$

we know at boundry $G(x,x') = 0$

So we left with only $\Phi(x) = \int \rho(x') G(x,x')$ where

$$G(x,x') = \frac{1}{|x - d\hat{i}|} - \frac{1}{|x + d\hat{i}|}$$



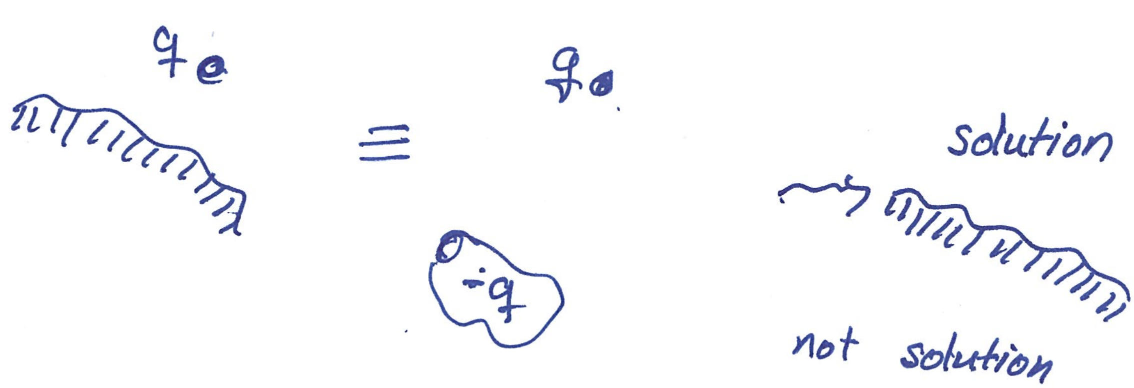
what would be the Green function ?!

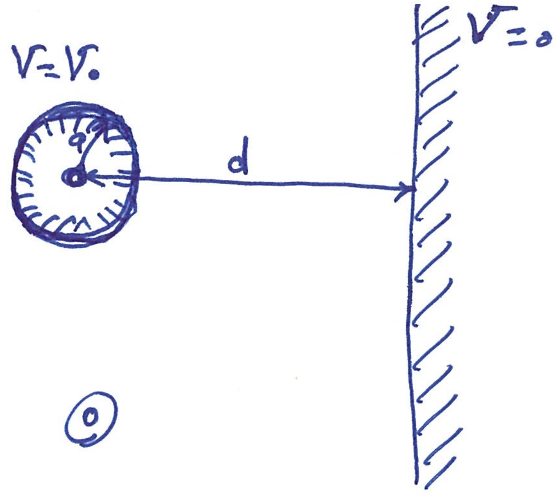
$$\Phi(x) = \frac{q}{|x-y|} - \frac{aq}{y\left|x - \frac{a^2}{y^2} y\right|} + \frac{Va}{|x|}$$

$$G(x,x') \sim \frac{1}{|x-x'|} - \frac{a}{x'\left|x - \frac{a^2}{x'^2} x'\right|}$$

now we can solve all sort of problems!

Q. where is the solution applicable ?!

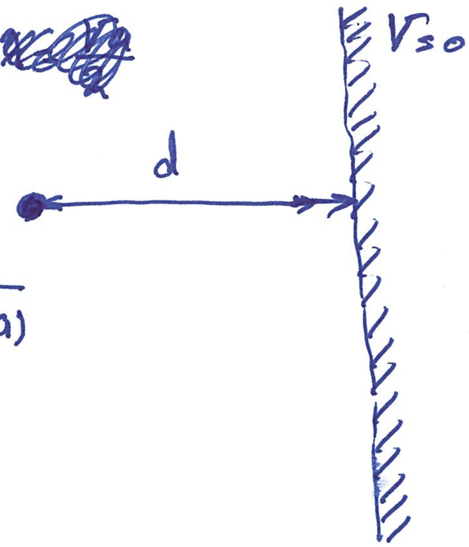




(0)

where

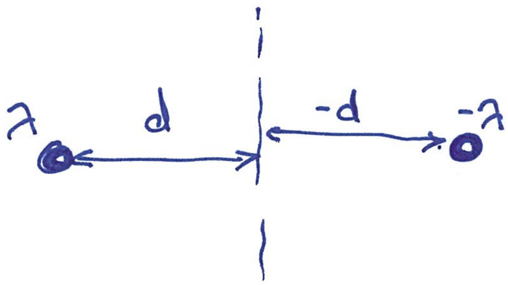
λ



$$\lambda = \frac{V}{2\ln(a)}$$

(1)

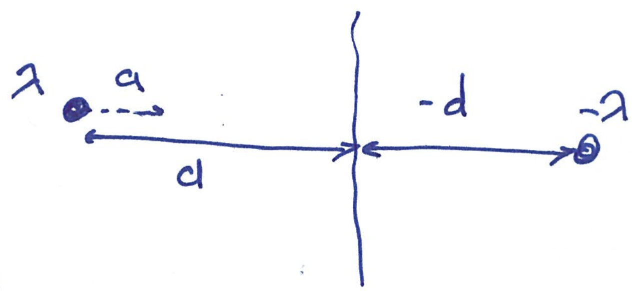
(2)



(3) Green function

we can write it.

Capacitance



$$\Delta V = \int_{-d+a}^{0} E\,dl$$

where $E(x) = \dots$

or we can calculat charge desity on cylinder.

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https://afarahi.github.io